



Tech Notes

Bits, Symbols, Noise, Modulation and Coding

The purpose of this memo is to clarify the relationship between the common factors used to determine performance of modern digital modems. Specifically I want to define bits and symbols, and the related digital signal to noise ratio numbers of energy per bit to noise density ratio (E_b/N_0) and energy per symbol to noise density ratio (E_s/N_0). The effect of coding and FEC will also be covered. Note that the following discussion concerns specifically digital transmission by BPSK and QPSK modulation using convolutional coding.

A "bit" is the basic information quanta in a transmission system. The noise density is the noise per Hertz averaged over the occupied bandwidth. The energy per information bit to noise density ratio is calculated as:

$$E_b/N_0 \text{ (dB)} = C/N_0 \text{ (dB)} - R \text{ (dB)}$$

Where: C/N_0 is the received carrier to noise density ratio in decibels, and R is the information rate expressed in decibels.

Note that this definition of E_b/N_0 includes no factor for the modulation method employed and in fact is independent of modulation.

We now define a new factor for what is actually transmitted from a modem with respect to the information (bit) rate. If we define a "symbol" as the basic transmitted quanta and the symbol rate as the maximum number of symbols that can be transmitted per second.

When coding is added to improve error performance the coding rate represents the ratio of transmitted information bits to transmitted coded bits (symbols). Thus a coding rate of $1/2$ represents that there are two transmitted symbols for each information bit.

The relationship between transmitted symbols and bits and noise density can be expressed by the formula:

$$E_b/N_0 \text{ (dB)} = E_s/N_0 \text{ (dB)} - r \text{ (dB)} ; \text{ where } r \text{ is the coding rate in dB.}$$

For example a coding rate of $1/2$ results in an r of -3 dB. Therefore the E_s/N_0 for this signal would be 3 dB less than the E_b/N_0 [$E_b/N_0 = E_s/N_0 - (-3\text{dB})$].

While modulation does not affect the bit rate or the E_b/N_0 , it can affect the symbol rate and E_s/N_0 . Modulation technique also affects the occupied bandwidth of the transmitted signal and consequently the Nyquist bandwidth used in the demodulator. Since QPSK modulates 2 bits of

information into each phase "state", while BPSK modulates 1 bit, QPSK modulation will occupy 1/2 the bandwidth of BPSK for the same data rate. If the same absolute output power is used to maintain the same E_b/N_0 at the receive end, the signal to noise ratio observed on a spectrum analyzer (with a measurement bandwidth less than the occupied bandwidth) will increase by 3 dB with QPSK modulation over BPSK. Note however that the E_b/N_0 is still the same. Therefore, to determine the E_b/N_0 of a signal by measurement with a spectrum analyzer it is necessary to know the modulation type used. Again since QPSK encapsulates two input symbols into each output symbol, the symbol rate of a QPSK system is 1/2 that of a BPSK system at the same data rate and E_b/N_0 , and the E_s/N_0 is 3 dB greater than that of the BPSK system.

The FEC coding determines a "coding gain" that is achieved over an uncoded system and can either reduce the power required for a constant error rate or improve the error rate performance at a given output power.

The symbol rate and E_s/N_0 is thus a factor of both coding and modulation. An example should illustrate these points. Assume that the same data at 64 kbps is to be transmitted over two channels, one BPSK and the other QPSK. Both will use either no coding or 1/2 rate coded FECs. The E_b/N_0 is actually a set point to achieve a certain error performance.

Data Rate	Modulation	Coding	Symbol Rate	E_b/N_0	E_s/N_0	C/N*	Nyquist BW
64 kbps	BPSK	1/1	64 ksps	6dB	6dB	6dB	64kHz
64 kbps	QPSK	1/1	32 ksps	6dB	9dB	9dB	32kHz
64 kbps	BPSK	1/2	128 ksps	6dB	3dB	3dB	128kHz
64 kbps	QPSK	1/2	64 ksps	6dB	6dB	6dB	64kHz

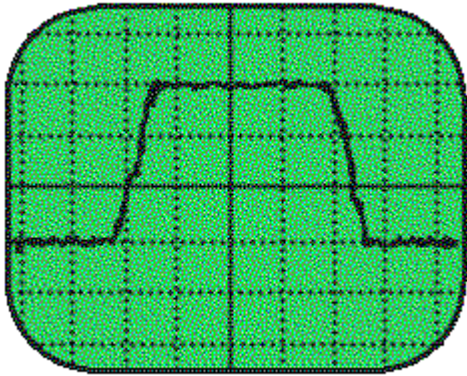
*the CN is the signal to noise ratio observed on a spectrum analyzer of the modulated carrier using a detection bandwidth much less than the signal occupied bandwidth. See note 2 below.

Spectrum Analyzer Measurement Notes

1. One potential problem is that when using the spectrum analyzer to measure signal to noise ratio the signal power should be integrated over the occupied band. With random modulation the displayed signal is fairly well integrated. If a V.35 scrambler is used to generate pseudo-random modulation with no input data, then the transmitted signal is missing a low frequency component due to the length of the scrambler's shift register and is no longer "rectangular" in shape. Instead it will be shaped like the top of a heart with two rounded lobes either side of a depression at the carrier frequency. Care must be taken here to integrate the signal power by eye to arrive at a true figure.
2. Another concern with analyzer measurement is that what is usually observed is actually the signal + noise to noise ratio, not the signal to noise ratio. For example if the observed signal + noise is 3dB above the noise level then the real signal to noise ratio is 0dB. Therefore the signal power must be measured in the absence of noise or computed from the signal + noise to noise ratio. Computation uses the formula:

$$S/N = 10\log(10^{((S+N)/N)} - 1)$$

In the spectrum analyzer display below, the S+N/N is approximately 6 dB (the scale is 2 dB per division). The SN is therefore: $10 \log(10^{6}-1)$ or 4.75 dB



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